Two Link Robot Arm

End point cost

$J := tf$

$$tf \tag{1.1}$$

State equations

$$eq1 := \frac{d}{dt}\,\omega 15(t) = \frac{1}{-189 + 100\cos(\theta 15(t))^2}\left(2\left(170\sin(\theta 15(t))\,\omega 16(t)^2 - 68\,T1(t)\right.\right.$$
$$+ 70\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t)) + 35\,\omega 15(t)^2\sin(\theta 15(t)) + 14\,T2(t)$$
$$+ 100\cos(\theta 15(t))\sin(\theta 15(t))\,\omega 16(t)^2 - 40\cos(\theta 15(t))\,T1(t)$$
$$+ 100\cos(\theta 15(t))\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t)) + 50\cos(\theta 15(t))\,\omega 15(t)^2\sin(\theta 15(t))$$
$$\left.\left. + 20\cos(\theta 15(t))\,T2(t)\right)\right)$$

$$\frac{d}{dt}\,\omega 15(t) = \frac{1}{-189 + 100\cos(\theta 15(t))^2}\left(2\left(170\sin(\theta 15(t))\,\omega 16(t)^2 - 68\,T1(t)\right.\right. \tag{2.1}$$
$$+ 70\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t)) + 35\,\omega 15(t)^2\sin(\theta 15(t)) + 14\,T2(t)$$
$$+ 100\cos(\theta 15(t))\sin(\theta 15(t))\,\omega 16(t)^2 - 40\cos(\theta 15(t))\,T1(t)$$
$$+ 100\cos(\theta 15(t))\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t))$$
$$\left.\left. + 50\cos(\theta 15(t))\,\omega 15(t)^2\sin(\theta 15(t)) + 20\cos(\theta 15(t))\,T2(t)\right)\right)$$

$$eq2 := \frac{d}{dt}\,\omega 16(t) = -\frac{1}{-189 + 100\cos(\theta 15(t))^2}\left(2\left(70\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t))\right.\right.$$
$$+ 35\,\omega 15(t)^2\sin(\theta 15(t)) + 14\,T2(t) + 35\sin(\theta 15(t))\,\omega 16(t)^2 - 14\,T1(t)$$
$$\left.\left. + 50\cos(\theta 15(t))\sin(\theta 15(t))\,\omega 16(t)^2 - 20\cos(\theta 15(t))\,T1(t)\right)\right)$$

$$\frac{d}{dt}\,\omega 16(t) = -\frac{1}{-189 + 100\cos(\theta 15(t))^2}\left(2\left(70\,\omega 16(t)\,\omega 15(t)\sin(\theta 15(t))\right.\right. \tag{2.2}$$
$$+ 35\,\omega 15(t)^2\sin(\theta 15(t)) + 14\,T2(t) + 35\sin(\theta 15(t))\,\omega 16(t)^2 - 14\,T1(t)$$
$$\left.\left. + 50\cos(\theta 15(t))\sin(\theta 15(t))\,\omega 16(t)^2 - 20\cos(\theta 15(t))\,T1(t)\right)\right)$$

$$eq3 := \frac{d}{dt}\,\theta 15(t) = \omega 15(t)$$

$$\frac{d}{dt}\,\theta 15(t) = \omega 15(t) \tag{2.3}$$

$eq4 := \frac{d}{dt}\,\theta16(t) = \omega16(t)$

$$\frac{d}{dt}\,\theta16(t) = \omega16(t) \tag{2.4}$$

$eqnl := [eq1, eq2, eq3, eq4]$

$$\left[\frac{d}{dt}\,\omega15(t) = \frac{1}{-189 + 100\cos(\theta15(t))^2}\left(2\left(170\sin(\theta15(t))\,\omega16(t)^2 - 68\,T1(t)\right.\right.\right. \tag{2.5}$$

$$+ 70\,\omega16(t)\,\omega15(t)\sin(\theta15(t)) + 35\,\omega15(t)^2\sin(\theta15(t)) + 14\,T2(t)$$

$$+ 100\cos(\theta15(t))\sin(\theta15(t))\,\omega16(t)^2 - 40\cos(\theta15(t))\,T1(t)$$

$$+ 100\cos(\theta15(t))\,\omega16(t)\,\omega15(t)\sin(\theta15(t))$$

$$\left.\left.+ 50\cos(\theta15(t))\,\omega15(t)^2\sin(\theta15(t)) + 20\cos(\theta15(t))\,T2(t)\right)\right), \frac{d}{dt}\,\omega16(t) =$$

$$-\frac{1}{-189 + 100\cos(\theta15(t))^2}\left(2\left(70\,\omega16(t)\,\omega15(t)\sin(\theta15(t))\right.\right.$$

$$+ 35\,\omega15(t)^2\sin(\theta15(t)) + 14\,T2(t) + 35\sin(\theta15(t))\,\omega16(t)^2 - 14\,T1(t)$$

$$\left.\left.+ 50\cos(\theta15(t))\sin(\theta15(t))\,\omega16(t)^2 - 20\cos(\theta15(t))\,T1(t)\right)\right), \frac{d}{dt}\,\theta15(t)$$

$$= \omega15(t), \frac{d}{dt}\,\theta16(t) = \omega16(t)\Bigg]$$

Initial and final conditions

$stateini := [\omega15(0) = 0, \omega16(0) = 0, \theta15(0) = 0, \theta16(0) = 0]$

$$[\omega15(0) = 0, \omega16(0) = 0, \theta15(0) = 0, \theta16(0) = 0] \tag{3.1}$$

$statefin := [\omega15(tf) = 0, \omega16(tf) = 0, \theta15(tf) = 1.761958473, \theta16(tf) = -0.09558107361]$

$$[\omega15(tf) = 0, \omega16(tf) = 0, \theta15(tf) = 1.761958473, \theta16(tf) = -0.09558107361] \tag{3.2}$$

Bounds and other data

$nstep := [[30]]$

$$[[30]] \tag{4.1}$$

$states := [\omega 15(t), \omega 16(t), \theta 15(t), \theta 16(t)]$

$$[\omega 15(t), \omega 16(t), \theta 15(t), \theta 16(t)] \tag{4.2}$$

$commands := [T1(t), T2(t)]$

$$[T1(t), T2(t)] \tag{4.3}$$

$t0bound := 0..0$

$$0..0 \tag{4.4}$$

$tfbound := 0..5$

$$0..5 \tag{4.5}$$

$commandbound := [-1..1, -1..1]$

$$[-1..1, -1..1] \tag{4.6}$$

$statebound := [-10..10, -10..10, -10..10, -10..10]$

$$[-10..10, -10..10, -10..10, -10..10] \tag{4.7}$$

$$commandsolini := \left[T1(t) = \begin{cases} 1 & t < 2.5 \\ -1 & t \geq 2.5 \end{cases}, T2(t) = \begin{cases} 1 & t < 2.5 \\ -1 & t \geq 2.5 \end{cases}\right]$$

$$\left[T1(t) = \begin{cases} 1 & t < 2.5 \\ -1 & 2.5 \leq t \end{cases}, T2(t) = \begin{cases} 1 & t < 2.5 \\ -1 & 2.5 \leq t \end{cases}\right] \tag{4.8}$$

Computations

$sol := OptimalControl\text{:-}Optimize(J, 0, eqnl, [\,], states, stateini, commands, statefin, t0bound, tfbound, commandbound, statebound, [\,], commandsolini, nstep, cache = \text{"TwoLinkRobotArm1"}, cache\_refresh = true)$

```
Optimal Solution Found.
```

$$\left[tf = 3.66776,\ t = \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ T1(t) = \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ T2(t) \right. \tag{5.1}$$

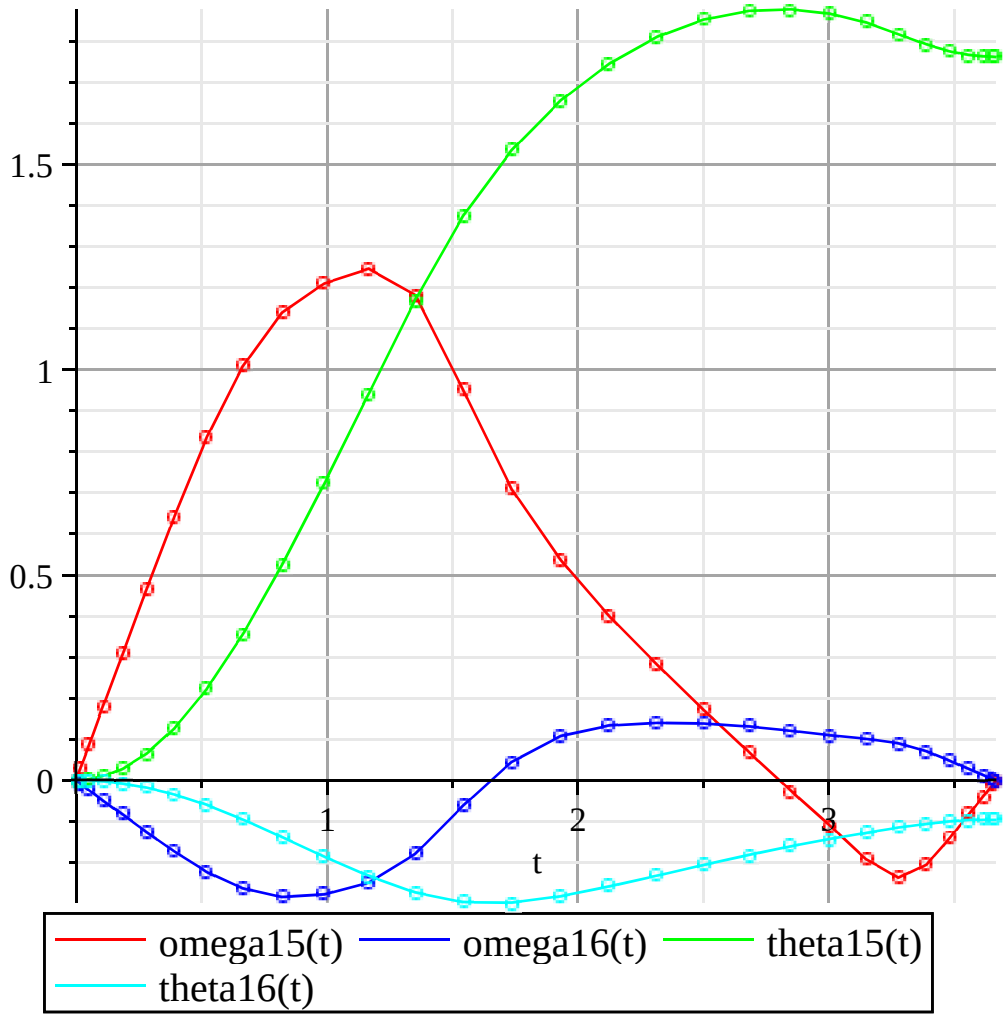
$$= \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ \omega 15(t) = \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ \omega 16(t)$$

$$= \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ \theta 15(t) = \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix},\ \theta 16(t)$$

$$= \left. \begin{bmatrix} 1 .. 30\ Vector_{column} \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix} \right]$$

Plots

OptimalControl:-*PlotSolution*(*sol*, [[*t*, $\omega 15(t)$], [*t*, $\omega 16(t)$], [*t*, $\theta 15(t)$], [*t*, $\theta 16(t)$]])



OptimalControl:-*PlotSolution*(*sol*, [[t, T1(t)], [t, T2(t)]])